

An Astonishing Regularity in Student Learning Rate

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Abstract

Leveraging a scientific infrastructure for exploring how students learn, we have developed cognitive and statistical models of skill acquisition and used them to understand fundamental similarities and differences across learners. Our primary question was why do some students learn faster than others? *Or do they?* We model data from student performance on groups of tasks that assess the same skill component and that provide follow-up instruction on student errors. Our models estimate, for both students and skills, initial correctness and learning rate, that is, the increase in correctness after each practice opportunity. We applied our models to 1.3 million observations across 27 datasets of student interactions with online practice systems in the context of elementary to college courses in math, science, and language. Despite the availability of up-front verbal instruction, like lectures and readings, students demonstrate modest initial pre-practice performance, at about 65% accuracy. Despite being in the same course, students' initial performance varies substantially from about 55% correct for those in the lower half to 75% for those in the upper half. In contrast, and much to our surprise, we found students to be astonishingly similar in estimated learning rate, typically increasing by about 0.1 log odds or 2.5% in accuracy per opportunity. These findings pose a challenge for theories of learning to explain the odd combination of large variation in student initial performance and striking regularity in student learning rate.

Significance statement

Prior research, often using self-report data, hypothesizes that the path to expertise requires extensive practice and that different learners acquire competence at different rates. Fitting cognitive and statistical growth models to 27 datasets involving observations of learning and performance in academic settings, we find evidence *for* the first hypothesis and *against* the second. Students do need extensive practice, about 7 opportunities per component of knowledge. Students do not show substantial differences in their rate of learning. These results provide a challenge for learning theory to explain this striking similarity in student learning rate. They also suggest that educational achievement gaps come from differences in learning opportunities and that better access to such opportunities can help close those gaps.

Introduction

Humans are capable of a wide and flexible variety of learning adaptation. This adaptability is particularly apparent in the development of expertise associated with high-profile careers, like technology innovation or music composition, but also in the wide variety of academic subject matter, reading, writing, math, science, second language, etc., humans master. Better understanding of how human learning works in the context of academic courses is of scientific interest because academic learning is particularly distinct to the human species. It is also of practical interest because such understanding can be used to develop more effective education. New technologies have often made better science possible. Such is the case for educational technologies which, in this century, have been increasingly providing unprecedented volumes of detailed data on academic learning. With center-level funding from the National Science Foundation to LearnLab (learnlab.org), we developed a social-technical infrastructure to systematically acquire such data and use it both to optimize interactive learning technologies and to pursue scientific questions about student learning.

LearnLab's early goals were to identify the mental units of learning in academic courses, to use these insights to design and demonstrate improved instruction in randomized controlled experiments embedded in courses, and to build models of learners that may reveal significant similarities and differences across learners. Past research produced methods for discovering and validating improved cognitive models of the mental units students acquire in academic courses (e.g., Cen et al., 2006). These improved cognitive models were used to redesign course units and random assignment field experiments comparing student use of the redesign (treatment) with the original design (control) demonstrated enhanced learning outcomes (e.g., Liu & Koedinger, 2017a; Koedinger & McLaughlin, 2010). A key theoretical hypothesis of these cognitive models is that a decomposition of learning into discrete units, or *knowledge components*, produces predictions that can be tested against student performance data across different contexts and at different times. Investigations across multiple datasets support this knowledge component hypothesis (e.g., Cen et al., 2006; Koedinger et al., 2016).

In this paper, we combine these cognitive models with statistical growth models to explore significant similarities and differences across academic learners. Our research questions are:

1. *Practice needed*: How many practice opportunities do students need to reach a mastery level of 80% correctness?
2. *Initial performance variation*: How much do students vary in their initial performance?
3. *Learning rate variation*: How much do students vary in their learning rate?

Research question 1 probes how much practice, if any, students need beyond the up-front verbal instruction they typically receive (e.g., from course lectures and/or reading assignments) before practice begins. We find that students are not at mastery at the start of practice and substantial learning occurs from the practice itself as students receive feedback on their performance and make use of context-sensitive verbal instruction and examples. We find that a typical student needs about 7 learning opportunities to master a typical knowledge component. However, we find substantial variation in needed opportunities across students. Thus, questions 2 and 3 probe whether that variation is more due to differences in initial performance and/or difference in the rate at which performance improves with each successive learning opportunity.

A rigorous and broad estimation of variation in student learning rate informs important scientific debates. Research on expertise (Ericsson et al., 1993) indicates that even historic geniuses needed years of practice to develop their expertise. Ericsson (Ericsson, 2008) estimates that high-level expertise takes about 10,000 hours of practice to develop and claims that no substantial exceptions have been found. In other words, no matter who you are, you need many repeated practice opportunities to develop expertise. As a counterpoint, other researchers (e.g., Macnamara & Maitra, 2019; Macnamara et al., 2016) have suggested that practice time alone is not enough to account for how much expertise is acquired and that some people may learn more (or less) than others given the same practice time. This debate comes down to whether learning rate per practice opportunity is relatively constant across individuals or whether it varies substantially. It has been difficult to make progress in resolving or refining this debate because of limitations in available data. Existing data fueling this debate comes from a small number of participants, is prone to subjective error as it is largely self-reported, and is sparse and coarse grained (few data points per participant). In contrast, the datasets we have accumulated directly track practice and are thus objective, are fine grained (about 200 observations per student), lasting over hours or weeks, and are large in students (nearly 7,000 students).

Beyond the deliberate practice debate, we find some researchers indicating substantial differences in student learning rate (National Research Council, 2002; Zerr et al., 2018) and others indicating little difference in student learning rate (Bloom, 1976). Consider, for instance, a National Academy of Sciences (NAS) report indicating that "high-ability learners learn at a more rapid rate than other students" (National Research Council, 2002, p. 131). In contrast to the National Academy of Sciences report, Bloom (1976) suggested that "most students become very similar with regard to ... rate of learning ... when provided with favorable learning conditions." (p. x). While Bloom and colleagues did demonstrate the effectiveness of a form of deliberate practice, they did not provide evidence to demonstrate their claim of uniformity in learning rate. Nor does the NAS report point to evidence for learning rate variability. This project provides an opportunity to test these competing claims.

Importantly, the NAS claim is about "high-ability learners", suggesting differences due to learner characteristics. It can be contrasted with a claim, which is almost certainly true, that learners in more favorable conditions learn at a more rapid rate than those in less favorable conditions. The educational technologies used in our NSF-funded LearnLab studies arguably provide favorable learning conditions as they implement research-based principles (e.g., varied practice with feedback and explanatory instruction) and many have been improved through iterative data-driven cognitive task analysis and experimental methods (Koedinger et al., 2012). A key goal of LearnLab was to identify, in the words of the NAS report, "high-ability learners" who "learn at a more rapid rate than other students" (*Pittsburgh Science of Learning Center Strategic Plan*, 2011). We were interested in identifying differences in students' self-regulated learning skills or background knowledge that would yield learning rate differences and that might be addressed through instructional support for learning to learn. Thus, we were quite surprised as results began to emerge suggesting an astonishing amount of regularity in student learning rate (Liu & Koedinger, 2017b).

One may be tempted by everyday experience to suggest there is obvious wide variability in how fast different people learn. At the end of an algebra course, for example, some students are getting an A and appear to have learned faster than other students who are getting low grades. However, such differences may be alternatively explained not as differences in learning rate but as differences in the number of quality learning opportunities individuals experience. In the varied data sources we have accumulated, the number of learning opportunities students experience is known and thus we can gain insight into whether student competence differences derive more from *environmental* opportunity differences or *student-inherent* learning rate differences.

In particular, we model learning using 27 datasets with over 1.3 million student performance observations from 6,946 learners in 12 different courses ranging across math, science, and language learning, across educational levels from late elementary to college, and across educational technologies including intelligent tutoring systems, educational games, and online courses (see Table S1 in Supplementary Information).

Should student performance captured in these datasets be considered representative of human learning generally? These datasets were produced by students using educational technology in natural contexts of academic courses. These courses involved common forms of instruction, such as lectures and assigned readings, which typically preceded student practice within the educational technology. While student practice has historically been done mostly on paper, we suspect, as in modern psychological experiments where participants interact with technology, that the technology interaction itself is not substantially changing the psychological

processes involved. Thus, these datasets are arguably well representative of complex human learning as it is displayed in academic contexts in math, science, and language learning.

Should these educational technology contexts be considered “favorable learning conditions” per Bloom’s claim? These contexts are prime examples of a learning-by-doing approach that has been repeatedly advocated in different variations and with substantial experimental support, including “mastery-based learning” (Bloom, 1968), “active learning” (Yannier et al., 2021), “testing effect” (Roediger & Karpicke, 2006), “formative assessment” (Black & Dylan, 2003), and “deliberate practice” (Ericsson et al., 1993; Macnamara et al., 2014; Koedinger et al., 2015). These contexts provide favorable learning conditions not only because of the active learning support, but also because of more particular features of learning interactions each with its own scientific basis. All these educational technologies (a) provide immediate feedback on errors in problem solving or performance contexts (Hattie & Timperley, 2007; Wisniewski, et al., 2020), (b) provide explanatory context-specific instruction on demand (e.g., Moreno, 2004), including an example correct response if needed (Crissman, 2006; Pashler et al., 2007; Salden et al., 2010), (c) highly encourage or enforce students to enter or observe a correct response before moving on, (d) provide tailored tasks designed through data-based cognitive task analysis to practice specific cognitive competences aligned with course goals for improving student thinking (e.g., Koedinger, Stamper, et al., 2013; Liu et al., 2014), and (e) give repeated opportunities to ensure student mastery of these cognitive competences (e.g., Ericsson, 1996) in varied tasks that require appropriate generalized, but not over-generalized, knowledge and skill acquisition (e.g., Paas & van Merriënboer, 1994).

Success \sim Initial-Knowledge + Learning-Rate * Opportunities
 \sim Overall + Student + KC + (Overall + Student + KC) * Opportunities

$$\ln\left(\frac{p_{ij}}{1-p_{ij}}\right) = \theta + \theta_i + \sum_{k=1}^K q_{jk} \beta_k + \sum_{k=1}^K q_{jk} (\delta + \delta_i + \gamma_k) T_{ik}$$

Cognitive model represented as matrix q_{jk}
indicates skill or concept components of knowledge k
needed to successfully perform & learn to perform task j

Figure 1. We model success (p_{ij}) of student i on task j across deliberate practice opportunities (T_{ik}) in a logistic regression with initial-knowledge and learning-rate estimates. These estimates are each decomposed into overall, student (i), and knowledge component (k) elements. The knowledge components required by each task (j) are specified in a cognitive model matrix (q_{jk}).

Modeling learning by integrating a cognitive model into a logistic regression growth model

To model student performance and learning, we used mixed effects logistic regression with a cognitive model component and a growth component. As indicated in the first line in Figure 1, we model the *success* of student i on task j as *proportional* to a linear function with an intercept that represents *initial-knowledge* (shown in orange) and a slope that represents *learning-rate* (shown in green) per *opportunity* to learn. Both initial-knowledge and learning-rate are further

broken down (second line in Figure 1) to provide an *overall* estimate and variations due to *student* and *knowledge component (KC)*. These lead to six parameters (predictor variables) in a mixed effects logistic regression (right side of equation in Figure 1) where the outcome variable (p_{ij}) is the probability that student i gets task j correct (left side but shown transformed by the log odds as per logistic regression). The six predictor variables include two fixed effects for overall initial-knowledge (θ) and overall learning rate (δ) and four random effects for the student initial-knowledge (θ_i), student learning-rate (δ_i), KC initial-knowledge (β_k), and KC learning-rate (γ_k) (see Supplementary Information for the precise model description in R). The three learning-rate parameters (in green) are multiplied by the number of opportunities (T_{ik}) student i has experienced on knowledge component k .

The product of learning rates by opportunity is the growth component of this model. Another key component is the cognitive model, which is represented by the matrix q_{jk} . This matrix indicates for each task j what knowledge component is needed to perform that task¹. In general, a cognitive model is an explanation of one or more cognitive processes that generates predictions that can be matched to human data (Busemeyer & Diederich, 2009). These cognitive models are implemented as computational models in datasets coming from Intelligent Tutoring Systems (see “ITS” in the Ed Tech column of Table S1 in Supplementary Information). The q_{jk} matrix provides a simplified representation of a cognitive model useful for statistical analysis. Many prior investigations have evaluated and refined these cognitive or KC models by comparing alternative versions of the q_{jk} matrix (Koedinger, Stamper, et al., 2013; Liu et al., 2014; Barnes, 2005; Demarais & Naceur, 2013; Stamper & Koedinger, 2011; Koedinger, et al., 2012). This KC model refinement process is illustrated in Table 1 with four different q_{jk} matrices for the same six tasks. As noted above, a task observation is often a step in a problem solution such that, for example, the problem 2*8-30 is observed in two steps as shown in the first two rows of Table 1.

Table 1. Four alternative cognitive models for the same tasks represented as q_{jk} KC models.

Tasks j (Observed Problem Steps)	Q0	Q1		Q2				Q3 = Item model					
	Arith	Mult	Sub	Mult LR	Mult OR	Sub+	Sub-	I1	I2	I3	I4	I5	I6
2*8-30 => 16-30	1	1	0	1	0	0	0	1	0	0	0	0	0
16-30 => -14	1	0	1	0	0	0	1	0	1	0	0	0	0
30-2*8 => 30-16	1	1	0	0	1	0	0	0	0	1	0	0	0
30-16 => 14	1	0	1	0	0	1	0	0	0	0	1	0	0
10-3*7 => 10-21	1	1	0	0	1	0	0	0	0	0	0	1	0
10-21 => -11	1	0	1	0	0	0	1	0	0	0	0	0	1

Note: Column groups Q0, Q1, Q2, Q3 represent different knowledge component (KC) models. Each row represents a task. 1 indicates that, in a given knowledge component model, the column KC is required in that task (row). For

¹The general statistical model allows for a task to be labeled by multiple KCs but such models tend not to produce better predictions than single KC models and the single KC models offer greater simplicity of interpretation.

example, for Q0, all tasks require arithmetic (Arith) whereas for Q2, tasks are hypothesized to require four different KCs depending on the operation involved and the context of its use.

Each alternative q_{jk} introduces different hypotheses about what makes tasks difficult (see the β_k term in Figure 1) and what yields transfer of learning across practice opportunities on related tasks (the term γ_k). Q0 reflects the hypothesis that all arithmetic tasks require one knowledge component (KC) and predicts all tasks will be of similar difficulty and practice on any one improves performance on another. Q1 separates multiplication and subtraction as different KCs. One may further hypothesize, as in Q2, that learning to solve tasks like the third ($30-2*8 \Rightarrow 30-16$) and the fifth requires extra order of operations knowledge (MultOR) whereas the first task ($2*8-30 \Rightarrow 16-30$) can be solved by a naive left-to-right strategy (MultLR). Q3 represents hypotheses that each task has its own inherent difficulty and that there is no transfer of learning across tasks. When q_{ik} is the identity matrix, as in Q3, the initial knowledge terms in Figure 1 are equivalent to item response theory (Wilson & de Boeck, 2004). Our model is similar to others who have used generalizations of item response theory to model student response data (e.g., Bergner et al., 2012; Pirolli & Wilson, 1998).

A KC model can be selected by comparing which q_{jk} matrix provides the best prediction fit to the student data. Care must be taken to use fitness measures that prevent overfitting due to increasing complexity (Q0-Q4) either by penalizing for greater parameters (e.g., AIC or BIC) or testing on held-out data via cross validation. DataShop facilitates such comparisons using a simpler version of the model in Figure 1 where the student learning-rate term (δ_i) is not included. We selected the best KC model for each dataset as discussed in Materials and Methods.

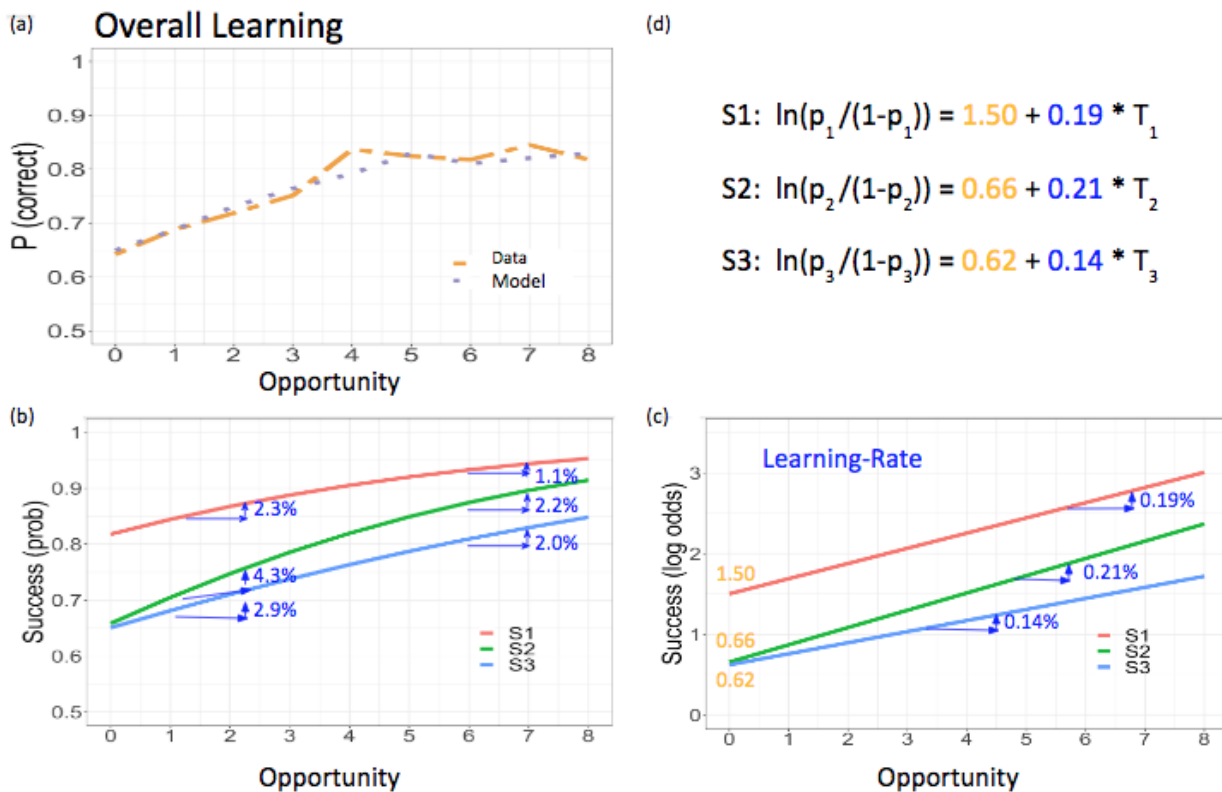


Figure 2. Example learning curves from dataset 394. (a) Learning curve and model predictions average over all students and KCs. (b) Model-based learning curves for 3 randomly selected students showing

non-linear percentage point slopes at two different opportunities. (c) Same curves in log odds scale with intercept values (yellow) and linear slopes (blue). (d) student i predicted success (p_i) at opportunity T as a function of intercept and slope.

Results

Illustrating and Evaluating a Learning Growth Model

Figure 2a shows a learning curve (in gold) from one of our datasets (ds394 in Table S1) where overall average probability correct (p_{ij}), on the y-axis, is increasing with successive opportunities (T_{ik}), on the x-axis. The model predictions are shown in green. Individual student curves derived from the model are shown in Figure 2b. It is difficult to visually compare student learning rates (e.g., is student S1 faster than S2?) in this non-linear scale where the opportunity-to-opportunity increase (e.g., 2.9% from opportunity 1 to 2 for S3) gets smaller at higher opportunities (2.0% from 6 to 7). Rescaling success using log odds, as shown in Figure 2c, produces a linear relationship whereby nuanced differences in student learning rate are apparent (e.g., S2 is steeper than S1 and S3). Figure 2d shows initial knowledge estimates (the intercepts in yellow) and learning rate estimates (the slopes in blue) for all three students, both in log odds.

We performed comparisons to evaluate the explanatory value of including student learning rate and measuring learning rate in terms of discrete opportunities rather than time. Much of the prior research in refining KC-based cognitive models (Cen et al., 2006; Cen et al., 2008) has used a simpler version of the model in Figure 1 where the student learning-rate term (δ_i) is not included. This simpler model has come to be called the Additive Factors Model (AFM)². The model in Figure 1 that includes individual student learning-rate parameters (δ_i) is called individual AFM or iAFM (Liu & Koedinger, 2017b). If iAFM provides a good model for detecting student learning rate variation, we should see that it outperforms AFM, at least on a substantial number of datasets. To be sure, such better performance of iAFM is neutral regarding the size of the student learning rate variation -- a topic we explicitly address further below.

AFM was the best fitting model in 6 out of 27 datasets based on AIC and 12 out of 27 datasets based on BIC, whereas iAFM was the best fitting model in 21 out of 27 datasets (AIC based) or 15 out of 27 datasets (BIC based). We used Wagenmakers (2007) approximation to derive Bayes factors from BIC. Across the 27 datasets, 15 had strong or greater evidence in favor of iAFM being the generating model ($BF > 1$), whereas 11 had strong or greater evidence in favor of AFM being the generating model ($BF < -1$). These results indicate that student learning rate variation is present and detectable in some datasets. In other datasets, it is either not present or too small to be detectable. As we elaborate later, even when student learning rate variation is detectable, it is not particularly large.

To evaluate the hypothesis that students learn as a result of KC-specific practice opportunities within the educational technologies, we contrast an additional model. The time-based Additive Factors Model (Time-AFM) implements the alternative hypothesis that students learn from general accumulated experiences in and outside the technology by using the elapsed calendar time to predict each performance observation. In this model, we replaced the count of practice opportunities used in iAFM and AFM (T_{ik}) with a calendar time variable. That is, to predict performance (p_{ij}) of student i on task j we use the calendar time C_{ik} that has passed

² The factors are “additive” because of the summation of log-odds values associated with KCs or “factors” in task difficulty and transfer. This addition of log-odds is in contrast with the multiplication of probabilities associated with KC factors in a more complicated Conjunctive Factors Model (Cen et al., 2008).

since this student first experienced knowledge component k associated with this task j . In Time-AFM, C_{ik} takes the place of T_{ik} in AFM. The results indicate that, despite its considerable overlap with AFM and iAFM, Time-Based AFM was only the best fitting model in 1 out of the 20 datasets (7 datasets did not have appropriate time-logging to run this comparison). Thus, we have clear evidence that learning growth is better characterized by KC-specific practice opportunities within the technology than by a calendar time variable that also reflects general opportunities for growth and for out-of-technology learning.

Students Start at about 65% Correctness and Need about 7 Practice Repetitions

Using the iAFM modeling results, we investigated students' typical initial performance. This investigation has relevance to the question of whether initial verbal instruction, most typically in the form of readings and lectures, is sufficient for reaching a reasonable level of mastery (defined as 80% correctness; Bloom, 1968). One possibility is that verbal instruction is enough for the average student to reach mastery and deliberate practice will just serve to strengthen what is learned, making performance faster and more fluent. Conversely, it is possible that verbal instruction is not sufficient to reach reasonable accuracy and deliberate practice is needed. As a measure of initial performance, we used the population iAFM intercept for each dataset (see θ in Figure 1). Table S1 shows the population intercept (initial knowledge estimate) for each dataset. The median population intercept (θ) across the 27 datasets is 0.638 log odds ($M = 0.501$, 95% $CI = [0.285, 0.718]$) which converts to 65.42% correct ($M = 61.79\%$, 95% $CI = [56.77\%, 66.82\%]$). That is, the typical student is starting practice well below mastery despite having been provided with verbal instruction in the form of readings and lectures prior to the deliberate practice experiences in our data.

Having established that with verbal instruction alone, students did not reach mastery, we investigated how much deliberate practice students tend to need. The median overall learning rate (δ) across datasets is 0.09 log odds ($M = 0.15$, 95% $CI = [0.08, 0.22]$) per opportunity, which converts to a 2.5 percentage point increase for one practice opportunity from the median intercept (θ) of 65%. We used the overall parameter estimates (θ and δ) from each dataset (with this formula $(\log\text{-odds}(.80) - \log\text{-odds}(\theta))/\delta$) to determine how many opportunities a typical student from that dataset would need to reach mastery. Across all datasets, the median number of opportunities to reach mastery is 7.24 ($M = 12.27$, 95% $CI = [7.09, 17.45]$). In other words, a typical student learning a typical KC tends to require seven additional practice opportunities to reach mastery *after* non-interactive verbal instruction (i.e., text or lecture).

Table 2. Median (standard deviations) across datasets for *initial accuracy* and *opportunities to reach mastery* for low (25) and high (75) percentiles of initial knowledge (assuming overall learning rate, δ) and learning rate (assuming overall initial knowledge, θ) for iAFM models.

Percentile	Initial Knowledge		Learning Rate	
	Initial % correct	Opp to reach 80% mastery	Improvement % correct*	Opp to reach 80% mastery
25	55.21% (15.84%)	13.13 (19.52)	1.70% (3.80%)	7.89 (22.41)
50	66.05% (12.91%)	6.54 (14.10)	2.25% (4.02%)	7.27 (14.18)

75	75.17% (10.45%)	3.66 (8.22)	2.56% (4.12%)	6.94 (10.91)
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Note: *Difference between initial % correct and first opportunity: Improvements are linear in log odds but get smaller in percent increase as performance approaches 100% correct.

Students Vary Substantially in Initial Knowledge

We investigated how much students vary in their initial knowledge using the model fits from iAFM. For each dataset, we computed the variance of student intercepts ($\theta + \theta_i$) and found the median of those to be 0.424 ($SD = 0.503$) and the interquartile range is 0.830 in log odds³. This large variation is more apparent if we compare the median student intercept of the lower and upper half of student intercepts. When converted to percentages, we see (first column of Table 2) that students in the lower half of initial knowledge had a median correctness of 55% and those in the upper half were 75% correct.

To highlight consequences of this substantial variability in initial knowledge, we compared estimated opportunities needed to reach 80% mastery for students in the bottom and top halves of initial knowledge (see the second column in Table 2). We used the same formula for computing opportunities given above but replaced the overall initial knowledge (θ) with the 25th and 75th percentile of the student initial knowledge estimates (θ_i). Whereas a student in the bottom half of initial knowledge needs about 13.13 opportunities to reach mastery, a student in the top half needs about 3.66 opportunities. In other words, a typical low initial knowledge student will take more than 3 times longer to reach mastery than a typical high initial knowledge student -- a large difference for students who have met course prerequisites and been provided verbal instruction.

Students are Astonishingly Similar in Learning Rate

Whereas initial knowledge varies substantially across students, we found learning rate to be astonishingly similar across students. This contrast can be seen in model-based student learning curves, like the one shown above in Figure 2c. The top of Figure 3 shows such curves for four datasets representing different course content, educational levels, and kinds of educational technology. See Supplementary Information Figures S6-S10 for KC and simulated data learning curves of all 27 datasets. Variation in initial knowledge is indicated by the wide range of intercepts in these curves. The similarity in student learning rate is illustrated by how generally parallel these curves are. While there are some cases of variation (e.g., see some non-parallel lines in the fourth panel for ds372), the log-odds increase in performance per opportunity is strikingly similar for most students in most datasets. This similarity in student learning rate is not only in contrast with much greater variation in student initial knowledge, but also in contrast to greater variation in knowledge component (KC) learning rates, shown in the middle of Figure 3. This learning rate variation by KC helps to alleviate a concern that we do not see variation in student learning rate because either our data or model are insufficient to detect such variation. The fact that we see substantial learning rate variation by KC and the obvious variation in the simulated student curves (bottom row of figure 3) indicates learning rate variations are detectable in these datasets with our model of learning, which relies on an empirically-refined cognitive model of domain competence inserted into a mixed effects logistic regression growth model.

³ We report parameter estimate variance using interquartile range (the difference between the 75th and 25th percentiles) to reduce dependence on distributional assumptions inherent in other measures of variance.

Low student learning rate variation is common across all the datasets as indicated by the mostly parallel lines in Figure 4 which shows the learning curves for the remaining 23 datasets. These curves also illustrate the much larger variation in student initial knowledge that is consistent across datasets. Note that the simulation results shown in Supplementary Information Figures S6-S10 (third column) confirm that our modeling approach can detect high student learning rate variation (and low student initial knowledge variation) when it is present.

To gain a statistical sense for relative variation in learning rate estimates, we calculated the variance and interquartile range of learning rate for students (δ_i) and KCs (γ_k) for each

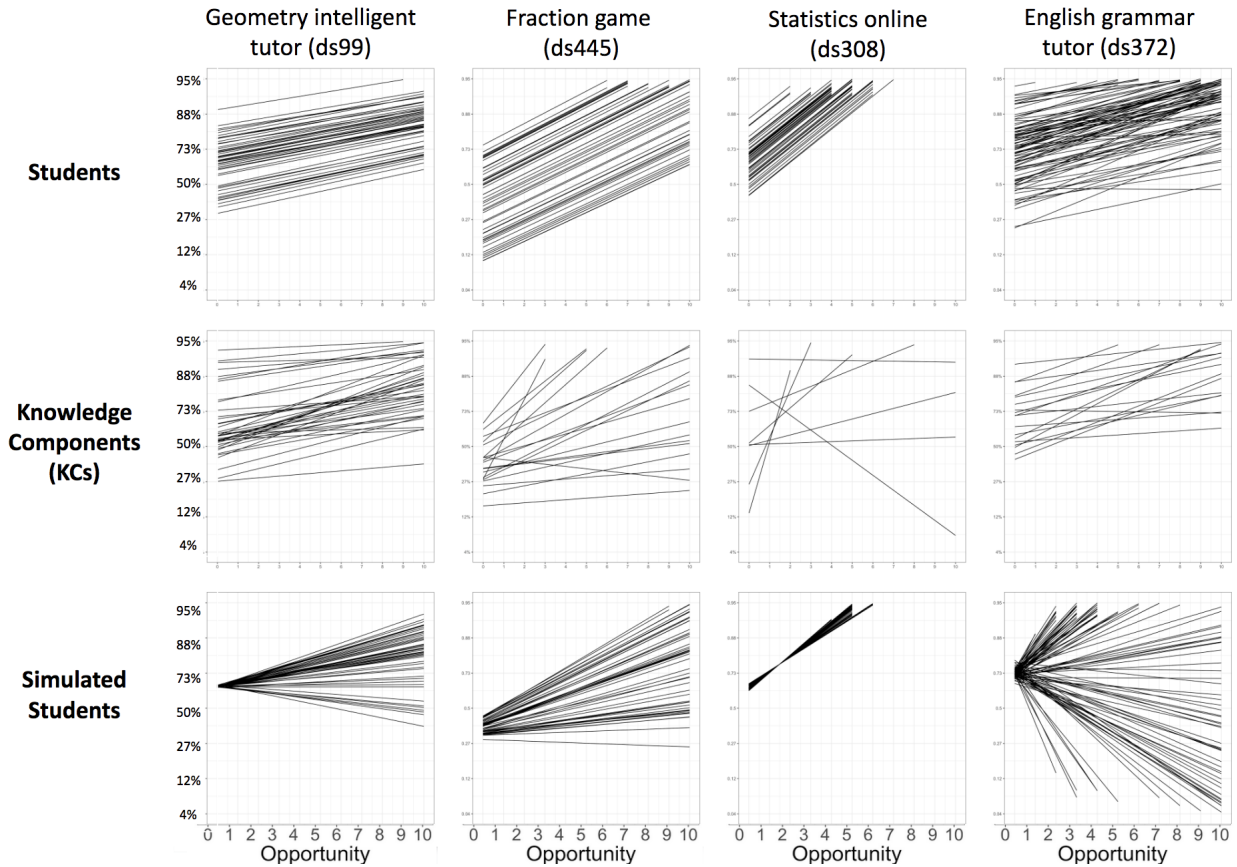
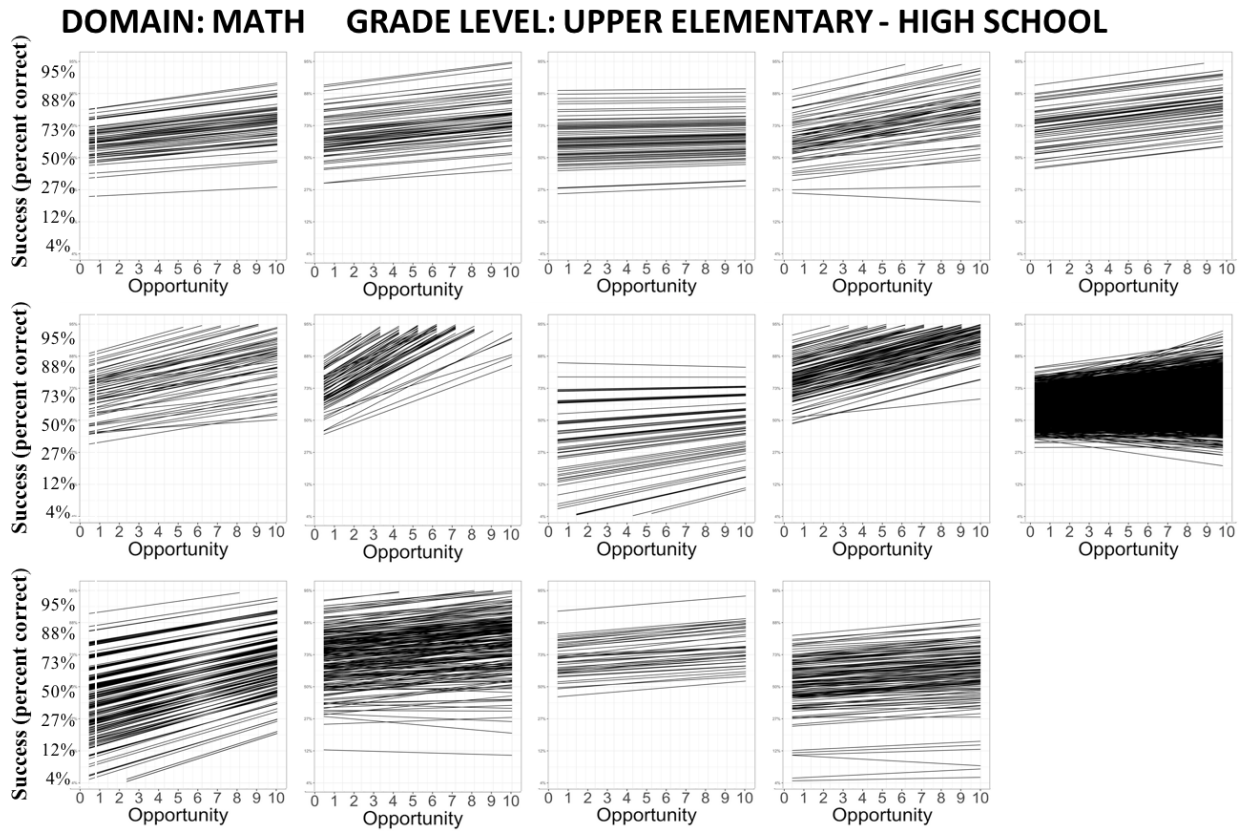


Figure 3. Learning curves relating opportunities to practice to performance accuracy in percent correct displayed on a log odds scale. The top graphs show student learning curves indicating little variation in student learning rate (i.e., lines are mostly parallel) in contrast to large variation in initial performance. The middle graphs show knowledge component (KC) learning curves indicating that learning rate variation is possible and measurable as these lines are not parallel. The bottom graphs demonstrate that the model can accurately identify high student learning rate variation and low student initial performance variation when they are known to be present as determined by simulation.

dataset. The median variance for student learning rate was 0.0002 ($M = 0.0008$; $SD = 0.0014$) and the interquartile range was 0.018 ($M = 0.028$, $SD = 0.024$) whereas the median variance for KC learning rate was orders of magnitude larger at 0.0104 ($M = 0.061$; $SD = 0.123$) and the interquartile range was seven times larger at 0.132 ($M = 0.175$, $SD = 0.147$). To help establish this low variance in student learning rate is robust and not, for example, only present when KC learning rate captures its variance, we investigated the impact of eliminating the KC learning rate parameters. We found that the individual learning rate estimates remain quite similar producing

little change in their variance with an interquartile range of 0.021 instead of the 0.018 log odds for the full model.

Returning to Table 2, we provide a concrete sense of the small variability of student learning rate relative to variability in students' initial knowledge. For columns 3 and 4 we divided students into groups based on percentiles of student *learning rate* estimates within each data set (whereas columns 1 and 2 are divided based on percentiles of student *initial knowledge* estimates). In percentage terms, the interquartile range in variation for student learning rate (see column 3) is only about 1% per opportunity (2.56%-1.70%), whereas the variation in initial



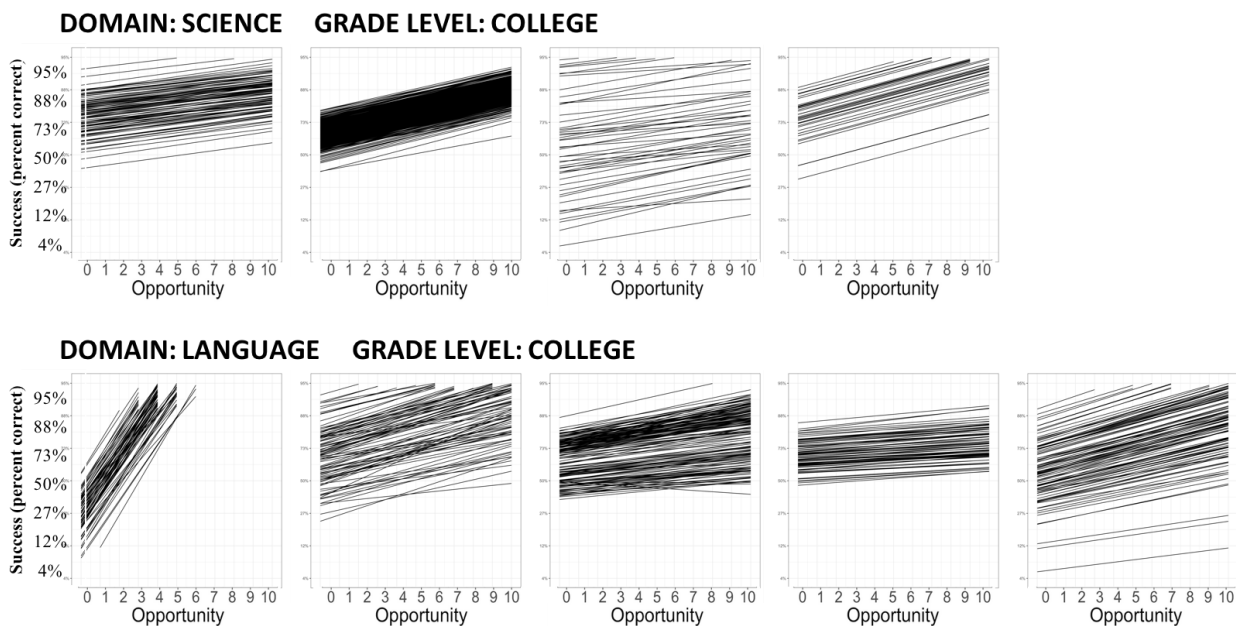


Figure 4. Across three domains, and various grade levels, little variation is observed in student learning rate but large differences are observed in student initial knowledge. These learning curves relate opportunities to practice to performance accuracy in percent correct displayed on a log odds scale.

knowledge is about 20% (75.17%-55.21%). We calculated for each percentile of student learning rate how many opportunities a student needed to reach mastery by subtracting the overall initial knowledge (θ) for each dataset from the mastery criteria (80% = 1.4 log odds) and dividing it by the median student learning rate parameter (δ_i) for that group of students (i.e., for each percentile of learning rate). Column 4 indicates that a typical student in the bottom half of learning rate (a slower learner) requires about 8 (Median = 7.89) opportunities to reach mastery whereas a typical student in the top half of learning rate (a faster learner) requires about 7 (Median = 6.94) opportunities. In other words, a typical slower learner needs only one extra opportunity to keep pace with a typical faster learner. In contrast, we observed much larger differences in initial performance, with the bottom half of initial performance being about ten opportunities behind the top half (13.13 - 3.66). The one opportunity difference to keep pace (i.e., span the interquartile range) in learning rate is an order of magnitude smaller than the ten-opportunity difference to catch up (i.e., span the interquartile range) in initial knowledge.

Perhaps there are particular circumstances in which higher student learning rate variation is revealed. For example, perhaps high initial performers cannot demonstrate their learning potential on easier KCs and higher learning rates would be revealed when we measure their rate on only the more difficult KCs. Conversely, low initial performers may show higher rates on easier KCs that are more within their reach. In fact, when we performed such an analysis we did not see such a pattern. The estimated student learning rates remained quite similar whether measured using only hard KCs, only easy KCs, or all KCs. The Supplementary Information provides more detail.

To further explore the possibility of high student learning rate variation in particular circumstances, we analyzed such variation disaggregated by domain (6 language, 16 math, and 5 science) and by student subpopulation based on grade level (8 elementary, 8 middle/high, 11 college). We use the interquartile range of the student learning rate hyper-parameter estimates

within each dataset and compute medians for the domain and grade level subgroups. Interestingly, there appears to be greater student learning rate variation in the six language domain data sets (median interquartile range of 0.045 log odds) than in the 16 math and five science domain datasets (median interquartile range of 0.017 log odds and 0.008 log odds, respectively). Student learning rate variation due to grade levels is more consistent with median interquartile ranges of 0.015 log odds for upper elementary, 0.018 log odds for middle/high school, and 0.018 log odds for college.

Discussion

We set out to use newly amassed fine-grained, longitudinal data to better understand the progress of academic learning. While hoping to create a method to identify high ability learners and understand their characteristics, we instead found strikingly similar rates of learning across students in context of the favorable conditions of interactive educational technologies. Along the way, we also demonstrated three other regularities. First, across a variety of courses, we found that initial practice performance is quite modest, about 65% correct (i.e., a failing grade), despite the general availability of up-front verbal instruction, such as lectures and readings. Second, we found that reaching a reasonable level of mastery (80% correct) requires substantial repeated practice, typically about 7 practice opportunities. These results are consistent with learning theories suggesting induction from examples and doing is prominent in human learning (Koedinger et al., 2012; Holland et al., 1989). Third, students' initial performance is highly variable despite students entering the courses in which the data was collected having met prerequisite requirements (for college courses) or age-level requirements (for K-12 courses) and having received verbal instruction.

Might the near constant student learning rate we observe be a consequence of limitations of our data or the measurement model? The first two results, that practice is needed and opportunity-based practice produces mastery, suggest our data has sufficient performance change to assess learning rate. The high variability in student initial performance indicates that our mixed effects growth model (iAFM) can differentiate individual student differences if present. The high variability in learning rate by knowledge component indicates that iAFM can differentiate learning rate differences if present. Thus, limitations in the data or measurement model do not appear to be good explanations for our near constant student learning rate observation.

Some readers may object that near constant student learning rate unrealistically implies that everyone can master advanced level calculus or interpret abstract data. Indeed, not everyone has favorable learning conditions nor will everyone choose to engage in the substantial number of practice opportunities required. However, our results suggest that if a learner has access to favorable learning conditions and engages in the many needed opportunities, they will master advanced level calculus. Other readers may object with intuitions of learning without substantial opportunities such as “I learned English without any practice” or “I learned calculus without attending lecture”. These claims may overestimate achievement and underestimate implicit learning-by-doing (i.e., speaking English or solving calculus problems) and informal learning outside of school (e.g., a discussion of calculus with a friend).

While our investigation of 27 datasets covers a wide variety of academic settings, it is possible that our results do not generalize to all academic situations that provide favorable learning conditions. Research supports the favorability of repeated practice with quality

feedback, as these educational technologies provide, however, there are forms of feedback such as peer interactions or classroom dialogue that are not well represented in our datasets.

Isolating Learning Rate Measurement

Intuitions that different students learn at different rates may derive from contexts that do not provide the same level of controlled investigation under favorable learning conditions that the interactive learning technologies helped achieve. The striking regularity in learning rate revealed here is not apparent if we do not control or account for other factors that drive student performance. We must isolate performance *changes* due to particular *opportunities* for learning that are *equivalent in nature and in number*.

One problematic conceptualization is to measure learning rate in terms of calendar time, such as 10 years to become an expert (Ericsson, 2008). While convenient, this conceptualization ignores that some individuals will have many more learning opportunities than others in the same period of time. By using learning data captured through interactive learning technologies, we are able to get a more *accurate accounting of learning opportunities* than has been previously possible. And indeed, we find these learning opportunities are much more predictive of learning outcomes than calendar time (a time-based model, time-AFM, systematically provides poor predictive fit).

A second problematic conceptualization of learning rate compares individuals getting different kinds of instructional support. To be sure, there is plenty of evidence that any of a wide variety of instructional interventions can greatly enhance student learning (e.g., Pashler et al., 2007; Koedinger, Booth, et al., 2013). However, such differences are a consequence of better instructional design not differences in students. Interactive learning technologies provide an effective way to control the nature of the learning opportunities that students experience. As driven by the materials and algorithms inherent in these technologies, all students are receiving the *same essential instructional interactions*.

A third problematic conceptualization is to merely compare individual performance overall without accounting for stable differences in performance due particularly to differences in prior experience. Our statistical modeling approach *accounts for stable differences* in performance by including an overall student performance latent variable as a per student baseline. We then measure learning rate as per-opportunity performance improvements above this baseline.

A fourth problematic conceptualization is to measure learning at the level of a broad topic or domain -- as though learning opportunities exercise a general faculty (e.g., “math” or “scientific reasoning”). Instead, we model and *measure learning in terms of finer grained components of knowledge*. This knowledge component (KC) modeling has been demonstrated to more accurately predict human learning data than a general faculty approach (Koedinger et al., 2016). Thus, critical to our approach was to identify datasets with KC models of reasonable quality whereby researchers have used empirical methods to evaluate and refine these KC models. In fact, when we fit the statistical model (iAFM) to the same datasets using less accurate KC models, estimates of overall learning rate goes down and estimates of student learning rate variation are even smaller (see Supplementary Information). This analysis indicates the importance of KC model refinement (Koedinger et al., 2016; Koedinger et al., 2012) in making student rate variation detectable. In fact, when we use a poor KC model, particularly the single KC model illustrated as Q0 in Table 1, the model with learning rate (iAFM) never outperforms the model without it (AFM).

Implications for Precise, Computational Theories of Learning

Taken together our findings pose a useful challenge for precise theories of human learning. Despite great progress in recent years, neither Cognitive Neuroscience nor AI has provided a full, precise account of how humans learn complex academic concepts and skills. Learning curves have been used as an inspiration for learning theory development, but past work focussed on modeling decrease in response times as a consequence of practice (Heathcote et al., 2000; Newell & Rosenbloom, 1981). Such theory might be sufficient if performance accuracy was easy to achieve and most learning occurred as speed-up in accurate performance. Our results suggest otherwise. Given our findings that a typical student starts practice-based learning at about 65% accuracy and that substantial practice, typically about 7 opportunities per knowledge component, are needed to achieve 80% accuracy, it is clear that learning theory must also account for changes in performance accuracy.

Our results suggest three specific theoretical challenges. A precise learning theory should explain and account for 1) continued changes in performance accuracy due to deliberate practice opportunities after initial up-front verbal instruction, 2) substantial student variation in initial performance, and 3) much smaller variation in student learning rate across practice opportunities. We briefly discuss each.

Explaining deliberate practice benefits. That up-front lectures and readings seem to produce limited performance accuracy is surprising given the great efforts educators continue to put into producing lectures and texts and given that most learners advocate explicit learning as the best way to learn (Carvalho et al., 2017). Books and then recorded lectures have facilitated broader dissemination of knowledge historically, but much emphasis on lecture recording remains today even in online course contexts where interactive practice is feasible and effective (cf., Koedinger et al., 2015). A theoretical postulate consistent with limited accuracy after up-front verbal instruction is that human learning is not simply about the explicit processing, encoding, and retrieval of verbal instruction but as much or more about implicit or non-verbal learning-by-doing in varied practice tasks where interactive feedback is available (Koedinger et al., 2012).

We observed that giving learners well-designed practice opportunities with feedback produces performance accuracy increases but learners typically require many such opportunities. Such interactive practice-based theory has support from empirical studies of expertise development (Ericsson et al., 1993) and experimental studies of the testing effect (Roediger & Karpicke, 2006) and active learning (Deslauriers et al., 2019). Theoretical models of human skill acquisition are generally consistent in the qualitative prediction that many learning opportunities are needed for a skill to be accurately acquired or a fact to be robustly recalled at a long interval (Laird, 2012; Anderson, 2009).

Explaining big differences in student initial knowledge intercepts. A class of general computational theories of cognitive skill acquisition, including ACT-R (Anderson & Lebiere, 1998), Soar (Laird, 2012), Icarus (Langley & Choi, 2006), Sierra (VanLehn, 1983) and Apprentice Learner (MacLellan et al., 2016), suggest that expertise develops through experience. This experience produces new skills stored in a procedural memory system made up of conditional knowledge components, typically implemented as if-then production rules. Such theories provide a straightforward explanation of big differences in student prior domain knowledge, namely, that some students have had more domain-relevant prior experiences than others. These past experiences produce domain-specific learning opportunities before the

window of observation within our data sets (e.g., discussion of fractions with a parent while making pancakes at home before fractions are covered in school).

This *prior-opportunities hypothesis* has been used to explain large intercept differences in *reaction time* learning curve data between children (with higher initial reaction times) and adults (with lower initial reaction times) given repeated practice on mental rotation tasks (Kail & Park, 1990). The learning curves of the two groups match under the single assumption that adults have had about 2000 more prior opportunities to practice than the children. The Apprentice Learner (AL; Weitekamp et al., 2020) theory has been used to support this prior-opportunities hypothesis in making accurate predictions of individual intercept differences in *error-rate* learning curves. As a fully functional computational model of learning, AL learns from tutoring interactions like those in provided by the interactive educational technologies used to produce our datasets and thus makes these predictions with only a single parameter per student representing unobserved prior practice. Unlike statistical models (e.g., Heathcote et al., 2000; Li et al., 2013; Steyvers et al., 2019), AL does not need parameters for knowledge component difficulty and learning rate because such differences are emergent from declining performance errors produced by the learning mechanisms inherent in it (e.g., learning *how* to do things to produce the then-part of production rules and learning both *where* to get needed information and *when* to do things to produce the if-part of production rules).

The *prior-opportunities hypothesis* suggests a concrete, though challenging, empirical test: If researchers can track and count domain-relevant learning opportunities that students experience prior to course entry, they should find a) large differences across students in these prior opportunities and b) that these differences substantially account for large initial performance differences at the start of within-course practice that we have observed.

Toward explaining small differences in student learning rate. We can infer a prediction of student learning rate variation from the learning mechanisms posited in prior skill acquisition theories like ACT-R, Soar, etc. Consider the compilation learning mechanism in ACT-R which posits that domain-specific skills are acquired from pre-existing domain-specific declarative knowledge and from pre-existing domain-general procedural knowledge which interprets these declarative memories to form new domain-specific skills. If we assume individual variation in the quality or quantity of this domain-general *background* knowledge, it follows from these skill acquisition theories that we should observe individual differences in learning rate. Similar arguments follow from Soar's chunking mechanism (Laird et al., 1986) and AL's inductive learning mechanisms (MacLellan et al., 2016).

Such *background knowledge* is used indirectly in the domain as support to answer questions (e.g., negative number concepts in equation solving) or as part of processing learning materials (e.g., reading skills for comprehending solution directions in equation solving). It can be distinguished from *prior domain knowledge*, which is used directly to answer questions or perform reasoning steps in the instructional domain (e.g., adding to both sides in algebra equation solving). Large differences in prior opportunities, it would seem, should not only produce differences in prior domain knowledge, as discussed above, but also differences in background knowledge. In turn, differences in background knowledge should produce differences in learning rate (cf., Delahay & Lovett, 2019; Li et al., 2010).

While we did find large differences in initial knowledge, we found quite small differences in learning rate. Why might that be? We suggest a *disjunctive learning path hypothesis* based on our observations of learning processes of the Apprentice Learner (AL) and consistent with skill

acquisition mechanisms in ACT-R and Soar. AL specifies a mechanism of academic learning whereby learners use background knowledge to search for and induce mental derivations or explanations of examples they experience (cf., Atkinson et al., 2003; Mitchell et al., 1986). To be sure, these mental explanations are modeled and conceived of as mostly non-verbal, inductive brain processes (cf., Koedinger et al., 2012) not explicit verbal reasoning that the term “explanation” may evoke. AL produces many alternative explanations of the same example steps especially when simulated learners are given different subsets of that background knowledge. Thus, AL predicts that, with different sampling of background knowledge, student learning produces differences in mental representation but similar performance outcomes. For example, one student may have existing background knowledge to self-explain an algebra example involving negative number subtraction whereas another student does not but compensates with a conceptual strategy using a number line to self-explain the same example.

Consistent with this *disjunctive learning path hypothesis*, a neuroimaging study demonstrated that students achieved equivalent performance in math problem solving with quite different mental representations (Sohn et al., 2004). Students instructed with a verbal representation solved problems as effectively but with higher activation in the left-prefrontal cortex than students instructed through a symbolic representation who revealed higher activation in the bilateral parietal cortices.

The higher learning rate variation we observed in the language datasets than in the math and science datasets also appears consistent with the *disjunctive learning path hypothesis*. The math and science domains allow for multiple learning paths in that the subject-matter includes generalized skills and re-discoverable principles and fewer verbatim facts than the language domains (cf., Koedinger et al., 2012). Learning in language domains is thus more dependent on rote memory to acquire arbitrary mappings (e.g., in English, oceans are referenced using “the” whereas lakes are not). Thus, variations in rote memory processing may produce greater learning rate variation in those domains than in math and science domains where rote memory limitations can be supplemented or compensated with general skill induction or sense making processes.

More generally, our educational system may be reasonably uniform in providing students with sufficient background knowledge for learning such that, for example, students enter an algebra course with enough background knowledge of integers, rationals, and arithmetic to learn from good examples, practice, and instructional feedback. Moreover, given favorable learning conditions, student learning may be substantially robust to small gaps in background knowledge. With good instruction, such as the quality deliberate practice interactive learning technology systems provide, students can compensate during learning for some gaps. We suggest further theory development and learning curve modeling to test these hypotheses.

Practical Implications

The learning rate question is practically important because it bears on fundamental questions regarding education and equity. Can anyone learn to be good at anything they want? Or is talent, like having a “knack for math” or a “gift for language”, required? Our evidence suggests that given favorable learning conditions for deliberate practice and given the learner invests effort in sufficient learning opportunities, indeed, anyone can learn anything they want. If true, this implication is good news for educational equity -- as long as our educational systems can provide the needed favorable conditions and can motivate students to engage in them. The variety of well-designed interactive online practice technologies used to produce our datasets point to a scalable strategy to provide these favorable conditions. Importantly, these technologies were well

engineered to provide the key features of deliberate practice including well-tailored task design, sufficient repetition in varied contexts, feedback on learners' responses, and embedded instruction when learners need it. At the same time, students do not learn from these technologies if they do not use them. Recent research providing human tutoring to increase student motivation to engage in difficult deliberate practice opportunities suggests promise in reducing achievement gaps by reducing opportunity gaps (Guryan et al., 2021; Chine et al., 2022).

Materials and Methods

Datasets. This project was possible because of LearnLab's DataShop, the world's largest repository of student learning data (Koedinger et al., 2010). We used 27 datasets (see Table 2) from DataShop that include an assortment of domains (e.g., geometry, fractions, physics, statistics, english articles, chinese vocabulary), of educational levels (e.g., grades 5-12, college, adult learners), and of settings (e.g., in class vs. out of class as homework). In general, students worked at their own pace through course materials and received as-needed assistance in the form of hints and feedback. In many cases, a predetermined period of time was set for completing the work (e.g., one or more class periods).

Within these educational technologies, students perform tasks by answering questions or, in some cases, entering solutions to problems in a step-by-step fashion. All entries are either selected responses or short constructed responses that are automatically scored, sometimes with the help of AI algorithms. Whereas some student responses are to 4-choice multiple choice questions, most requested student responses are open. Such responses include text fields where students enter numbers (e.g., "72.3"), expressions (e.g., " $(972+b)/5$ "), or a word (e.g., the pinyin spelling of a Chinese symbol). They also include graphical user interface actions such as clicking a place on a number line (ds445) or drawing a force vector (ds104). Some multiple choice questions involve more than 4 options such as a list of some 12 possible explanations for an English article choice. Example tasks can be found in Supplementary Information (Figures S1-S4). Student responses to these tasks are automatically tagged as correct when students answer correctly on their first attempt without asking for a hint. Otherwise, the task response is tagged as an error. To estimate performance at a given task opportunity, only the student's first attempt is considered, though subsequent student attempts and system feedback are critical contributors to learning. We define learning as a positive change in performance and operationalize learning as a reduction in error rate (or increase in correctness rate) over successive opportunities to perform a task associated with a specific knowledge component.

Dataset Selection Criteria. Among hundreds of datasets available in DataShop, we identified 27 datasets to include in our analysis (see Supplementary Materials tables S1 and S2). To achieve accurate parameter estimation from a dataset, it is critical to have a quality knowledge component (KC) model (cf., (Cen et al., 2006; Koedinger et al., 2016)). Thus, we looked for datasets where a KC model meets a set of precise criteria for quality and interpretability. To be included, a dataset must have an associated KC model that is better than at least two extreme alternatives (defined as lowest root mean squared error on the test set in three-fold item-blocked cross validation), an item-based model where each distinct task is coded as a different KC (e.g., Q3 in Table S1), and a faculty model where all unit tasks are coded as a single KC (e.g., Q0 in Table S1). For straightforward interpretation, we only considered KC models that involve a single KC label per unit task (see footnote 1). For datasets that had more than the three KC models implied above, we selected the KC model that had the best overall

prediction fit in item stratified cross validation (always comparing on the same sample of student observations).

We investigated the impact that the quality of the KC model had on the results we present. Better KC models tend to both increase overall learning rate (cf., (Koedinger et al., 2012)) and slightly increase student learning rate variance. Nevertheless, our main results, particularly low student learning rate variance, remain regardless of the KC model chosen (see Supplementary Materials for details).

We eliminated two datasets where the initial overall success rate was greater than 80% as these datasets leave less room to observe learning. Among the selected 27 datasets, we noticed that some specific KCs were too limited in number of associated data points. Thus, in each dataset, a KC was included only if there were data from at least 10 students with at least 2 learning opportunities. For 13 datasets, all KCs were included and in the other 14 datasets, an average of 2.7 of 952 KCs were excluded.

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Supplementary Information for
An Astonishing Regularity in Student Learning Rate

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This PDF file includes:

Supplementary text
Figures S1 to S11
Tables S1 to S5
SI References

Supplementary Text

Model notation in R. Below are our iAFM, AFM, and time-AFM models as implemented in R using package “lme4”:

```
iAFM: afm.model <- glmer(success ~ opportunity + (opportunity|individual) + (opportunity|KC), data=df, family=binomial())
```

```
AFM: afm.model <- glmer(success ~ opportunity + (1|individual) + (opportunity|KC), data=df, family=binomial())
```

```
time_AFM: afm.model <- glmer(success ~ timeDiff + (1|individual) + (timeDiff|KC), data=df, family=binomial())
```

Simulated data. To test whether the models we developed would be able to detect learning rate variability if it were present, we created simulated data and refit the models. We manipulated the fitted student intercept (the 1|individual term above) and slope (opportunity|individual) hyper-parameters from iAFM to produce new hyper-parameters to generate 3 new simulated datasets (with the same total rows of student-task observation) with higher slope variability and/or lower intercept variability. In particular, we multiplied the individual slope hyperparameters (which are centered new near 0) by 10 to produce higher greater student learning rate variability and/or divided the individual intercept hyperparameters by 10 to produce smaller student prior knowledge variability. Each simulated dataset generated new student-task correctness (for all the same student-tasks in the original dataset) by using the modified hyperparameters and unmodified KC hyperparameters (opportunity|KC) and overall slope parameter (opportunity) to make model predictions which were randomly converted to 0 or 1 (using the function “rbinom” in R). The 3 new data sets complete a 2x2 with respect to low vs high variability in student intercept or student slope. In all cases, the iAFM model well recovered the true parameters finding a high student slope and/or low student intercept (as illustrated in Figures in the main paper and below).

Impact of KC model quality. For the analyses described in the main text we selected the best KC model for each dataset. However, it is possible that the KC model selected, and how well it maps to the knowledge being acquired, has an impact on the results. For example, it is possible that the small variability in student learning rate is due to a poor KC model (even if the best KC is selected, it could theoretically still not be a good match). We investigated this possibility in two complementary ways. First, for all datasets, we compared the chosen KC model with the single KC model (a model where every step is considered the same KC, see Q0 in Table 1 in the main text). Second, for one of the datasets for which there are a substantial number of KC models (dataset 99), we compared the student learning rate variability across KC models, ranked from best to worst using the same metric used to select the best KC models to use in the main analyses.

Overall, for all datasets but 2 (see Table S4 below), student learning rate variability, as measured by IQR, was higher for the best-KC model ($M = 0.028$, $Median = 0.018$, $SD = 0.024$) than the single KC model ($M = 0.005$, $Median = 0.002$, $SD = 0.007$). Similarly, as shown in Figure S11 below, student learning rate variability increases with increased quality of the KC model ($r^2 = .55$, $p = .029$).

These make two important contributions. On the one hand, regardless of KC model quality, student learning rate variability is consistently small. On the other hand, it does increase with the quality of the KC model, arguing for the importance of KC model refinement for appropriate student modeling ((Koedinger et al., 2016)).

Easy vs. Hard KCs. One potential risk in our estimation of learning rate is that students who start with high prior knowledge may have little room to grow and thus their true learning rate may be

underestimated. This ceiling effect issue is most likely to show up on easier knowledge components and may be reduced or eliminated if we estimate learning rate just for the harder knowledge components. We suggest this strategy for future research.

To check, we used the iAFM intercept estimates for each KC to identify easier (below median) and harder (above median) KCs. We then fit two modified iAFM models for each dataset, one using only data from the easier KCs and another using only data from the harder KCs. To isolate learning rate estimates and eliminate variations in other estimates across the two datasets, the modified iAFM model used a student slope parameter to predict performance offsetting the prediction with fixed iAFM estimates for student intercept, KC intercept and KC slope.

As shown in Table S5 below, median variability in student learning rate, as measured by interquartile range (IQR) was similar for high and low ability students learning easy or hard KCs. IQR varied from 0.034 for lower ability students learning easy KCs to 0.066 for higher ability students learning hard KCs.

ANDES Physics Workbench - [s2e-Solution]

File Edit Diagram Variable View Help

A spherical ball with a mass of 2.00 kg rests in the notch shown below. If there is no friction between the ball and the walls, what is the magnitude of the force exerted on the ball by wall1?

Answer:

Variables:

Name	Definition	Dir
T0	the instant depicted	
m=2 kg	mass of ball	
x	axis	$\theta_x=0^\circ$
Fg	magnitude of the Weight Force on...	$\theta_{Fg}...$
F1	magnitude of the Normal Force on...	$\theta_{F1}...$
a	magnitude of the Instantaneous A...	

1. $F_{g,y} + F_{1,y} = 0$

2.

3.

4.

5.

6.

7.

8.

9.

T: There is a force acting on the ball at T0 that you have not yet drawn.
Explain further OK

T: Notice that the ball is supported by a surface: wall2.
Explain further OK

T: When an object is supported by a surface, the surface exerts a normal force on it. The normal

For Help, press F1

00:09:16 SCORE: 20

Fig. S1. Example screenshot from the physics intelligent tutor used in the context of a college Introductory Physics course to produce dataset ds104.

Ms. Lindquist #1

Ms. Lindquist is a math teacher. Ms. Lindquist teaches 62 girls. Ms. Lindquist teaches b boys. Write an expression for how many students Ms. Lindquist teaches.

[Comment on this question](#)

The number of students Ms. Lindquist teaches is equal to the number of boys plus the number of girls.

[Comment on this hint](#)

Show me the last hint

Type your answer below (mathematical expression):

Submit Answer

Ms. Lindquist #2

Ms. Lindquist is a math teacher. Ms. Lindquist teaches 62 girls. Ms. Lindquist teaches f fewer boys than girls. Write an expression for how many boys Ms. Lindquist teaches.

[Comment on this question](#)

The number of boys she teaches is equal to the number of girls minus the fewer boys.

[Comment on this hint](#)

Show me the last hint

Type your answer below (mathematical expression):

Submit Answer

Substitute $62-f$ for b in $62+b$
Write the resulting expression.

[Comment on this question](#)

Show me hint 1 of 3

Type your answer below (mathematical expression):

Submit Answer

Fig. S2. Example screenshot showing scaffolds and hints from the ASSISTments intelligent tutor on math symbolization used in the context of a middle school math class to produce dataset ds271.



Fig. S3. Example screenshot from the Battleship Numberline Answer-based tutor on fractions used in the context of an upper elementary math class to produce dataset ds445.


learn by doing

Bob needs to calculate the surface area of the legs of the tripod. (radius = 0.75 in. and height = 27 in) First he substitutes the values for radius, height and π into the equation.

$$SA = (2)(3.14)(0.75^2) + (2)(3.14)(0.75)(27).$$

What is the first thing Bob should do?

- Multiply $(2)(3.14)(0.75)$
- Simplify 0.75^2
- Add $0.75 + 2$
- Multiply $(2)(3.14)$



NEXT » Reset this Activity

did I get this

What is the surface area of the cylinder that will be used for the center column, rounded to the hundredths place?
(radius = 1.2" and height = 30")

- 226.08 in³
- 425.93 in³
- 74.88 in³
- 235.12 in³

Reset this Activity

Fig. S4. Example screenshot from the OLI Answer-based tutor on Stem Readiness used in the context of a college Developmental Mathematics course to produce dataset ds1935.

Compare the two sentences: Why do they require *different* articles?

1) Mount Kilimanjaro is the highest mountain in Africa.

Correct!

The rule to apply is: **0-single mountains, islands.**

Use **0** with the names of *single mountains and islands*.

Examples using the same rule:

Mount Olympus Bell **Mountain**

Mt. Everest Easter **Island**

Long **Island** Ellis **Island**

2) I want to go hiking in Appalachian Mountains.

Wrong.

The rule to apply is: **the-plural mountains, islands.**

Use **the** with the names of *plural mountains and islands*.

Examples using the same rule:

the Rocky **Mountains** **the** Ruby **Mountains**

the Alps (**mountains**) **the** Hawaiian **Islands**

the British **Isles** **the** San Juan **Islands**

Fig. S5. Example screenshot from English grammar tutor used in the context of college advanced English as a Second Language course to produce dataset ds447.

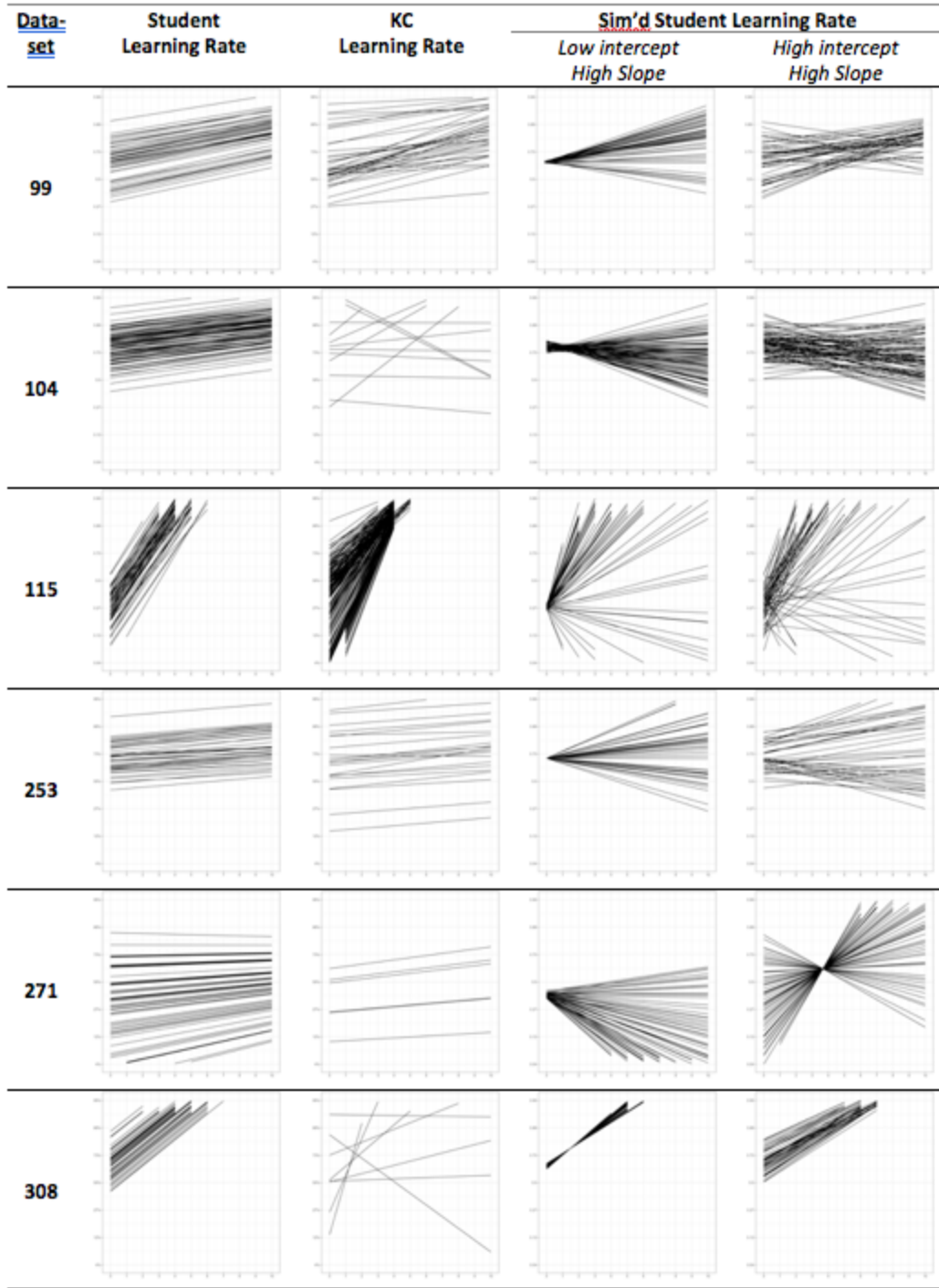


Fig. S6. Learning curves for 6 datasets with opportunities from 0-10 across the x-axis and percent correct on a log odds scale for the y-axis. Learning rate variation (slope) is relatively constant for students (near parallel lines in the first column) and higher for knowledge components (less parallel lines in the second column). The model can recover high student slope variation (non parallel lines) when present in simulated data whether the student intercept variation is low (column 3) or high (column 4).

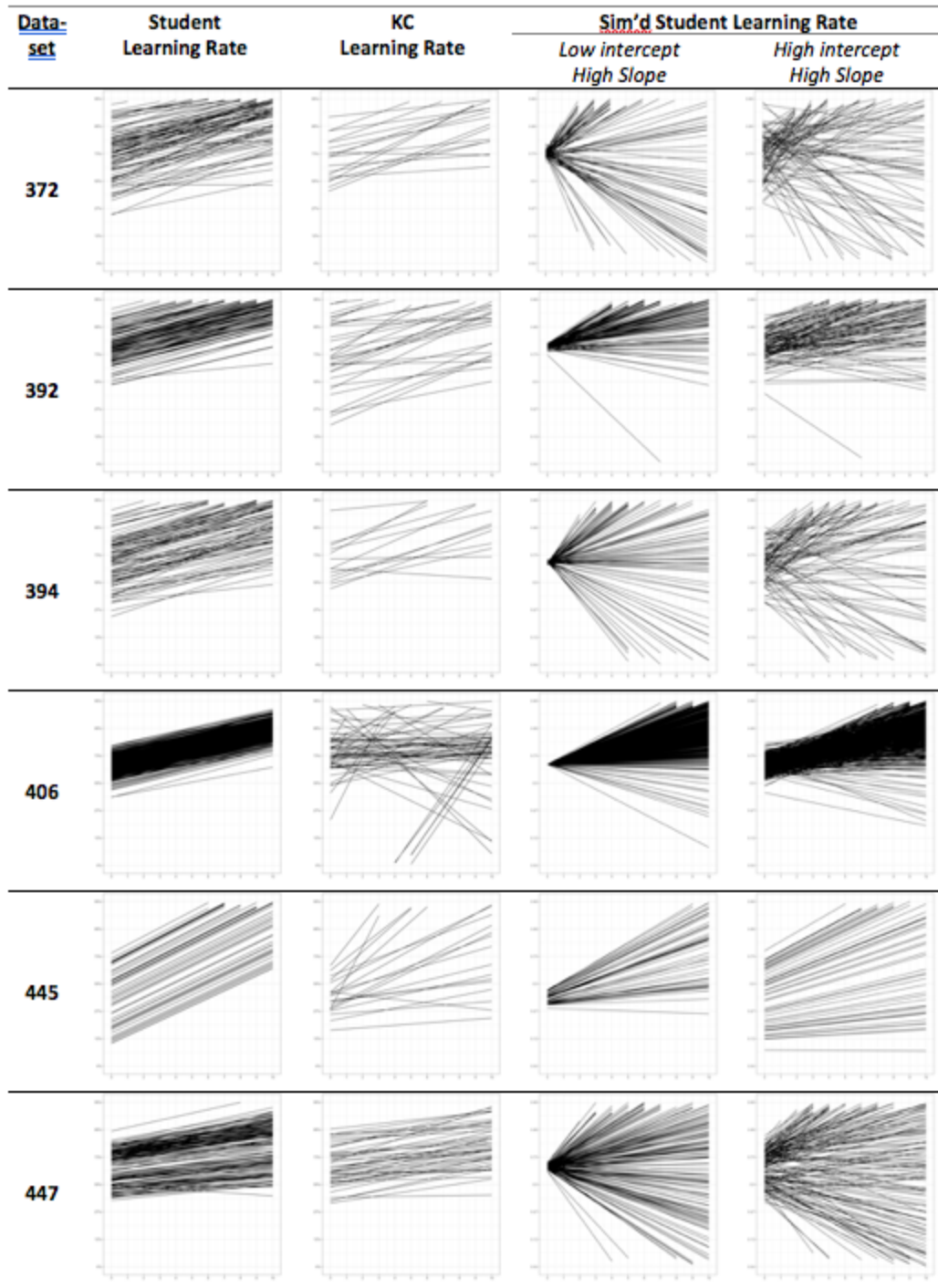


Fig. S7. Learning curves for 6 datasets with opportunities from 0-10 across the x-axis and percent correct on a log odds scale for the y-axis. Learning rate variation (slope) is relatively constant for students (near parallel lines in the first column) and higher for knowledge components (less parallel lines in the second column). The model can recover high student slope variation (non parallel lines) when present in simulated data whether the student intercept variation is low (column 3) or high (column 4).

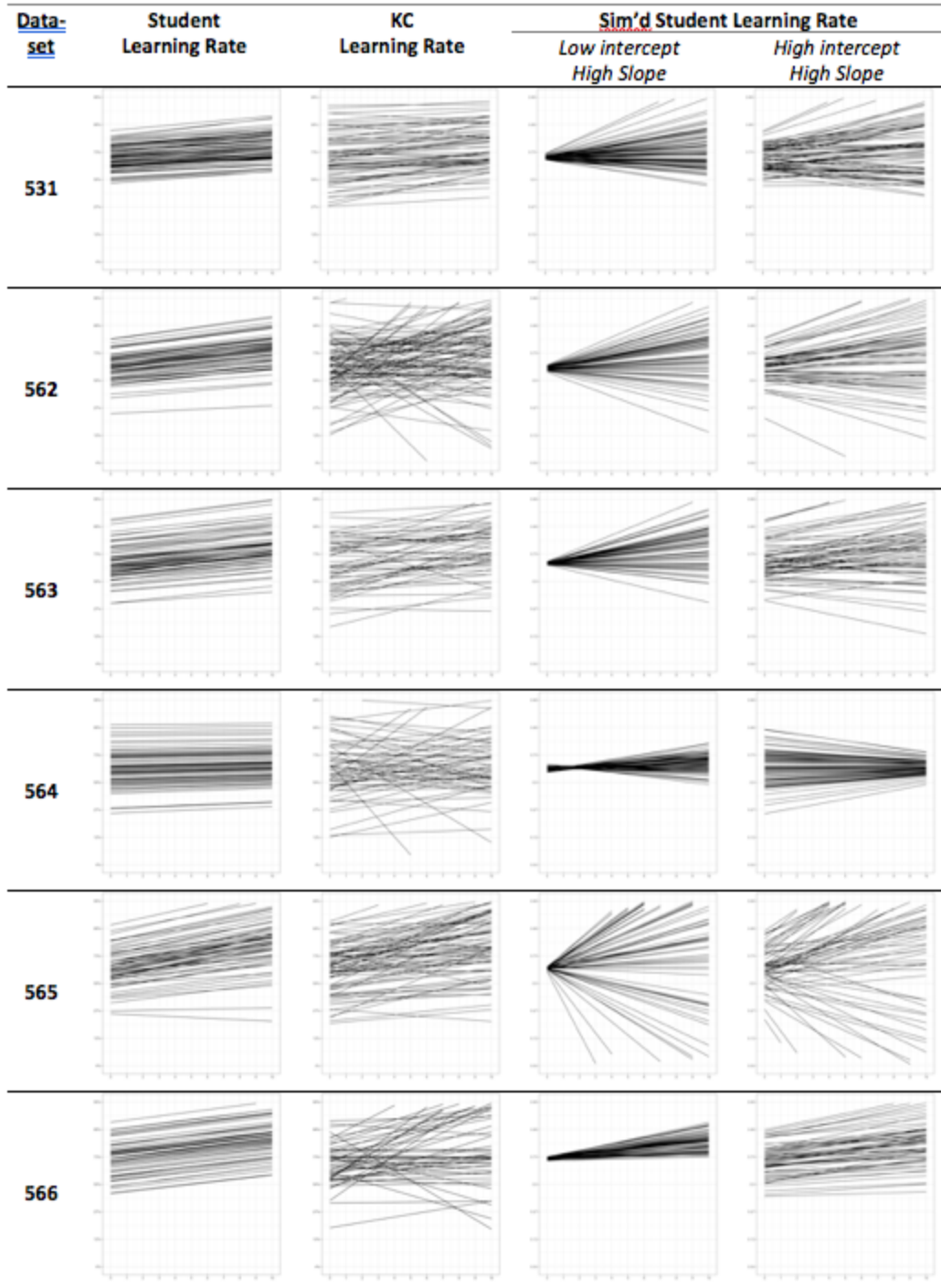


Fig. S8. Learning curves for 6 datasets with opportunities from 0-10 across the x-axis and percent correct on a log odds scale for the y-axis. Learning rate variation (slope) is relatively constant for students (near parallel lines in the first column) and higher for knowledge components (less parallel lines in the second column). The model can recover high student slope variation (non parallel lines) when present in simulated data whether the student intercept variation is low (column 3) or high (column 4).

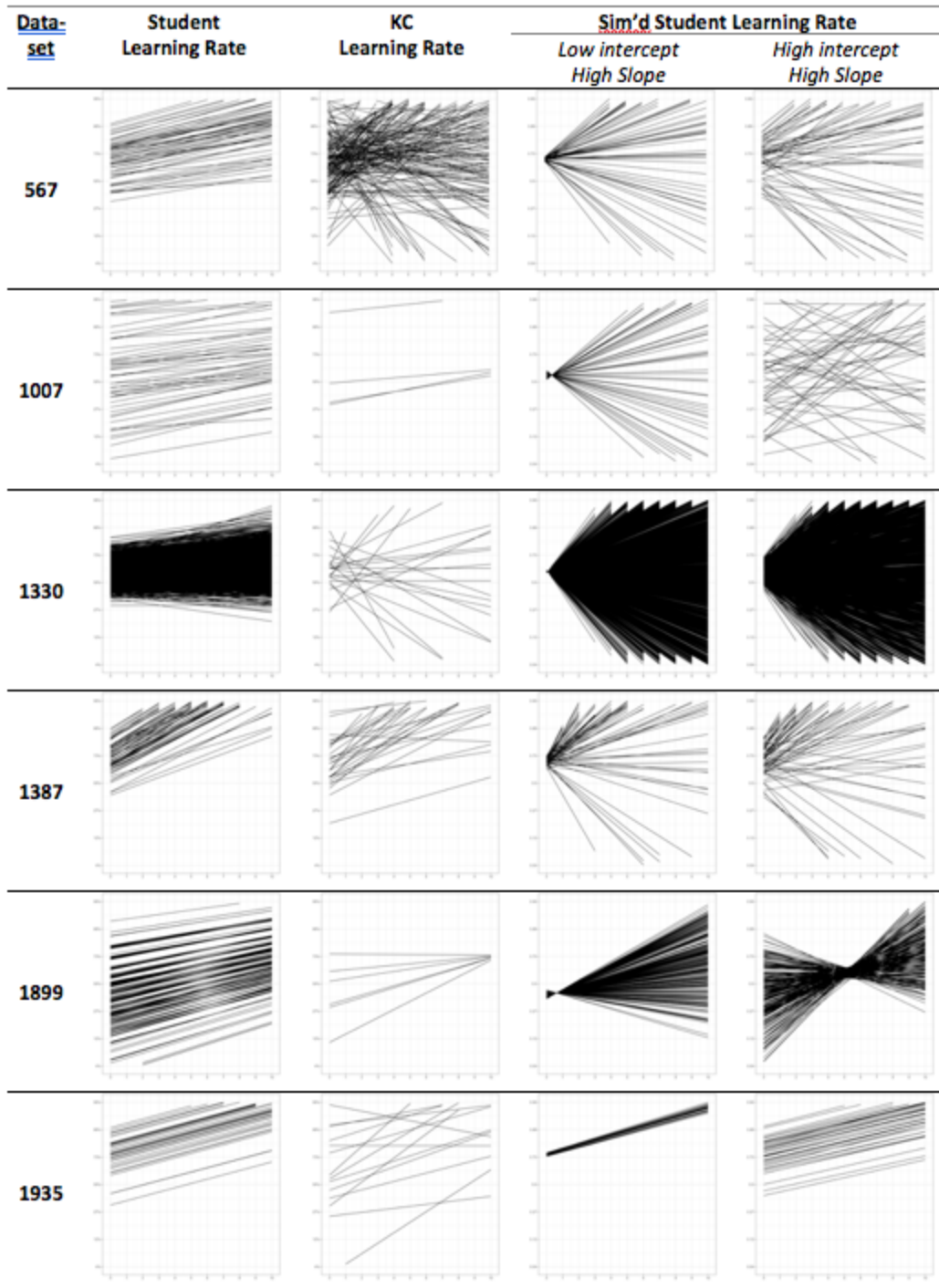


Fig. S9. Learning curves for 6 datasets with opportunities from 0-10 across the x-axis and percent correct on a log odds scale for the y-axis. Learning rate variation (slope) is relatively constant for students (near parallel lines in the first column) and higher for knowledge components (less parallel lines in the second column). The model can recover high student slope variation (non parallel lines) when present in simulated data whether the student intercept variation is low (column 3) or high (column 4).

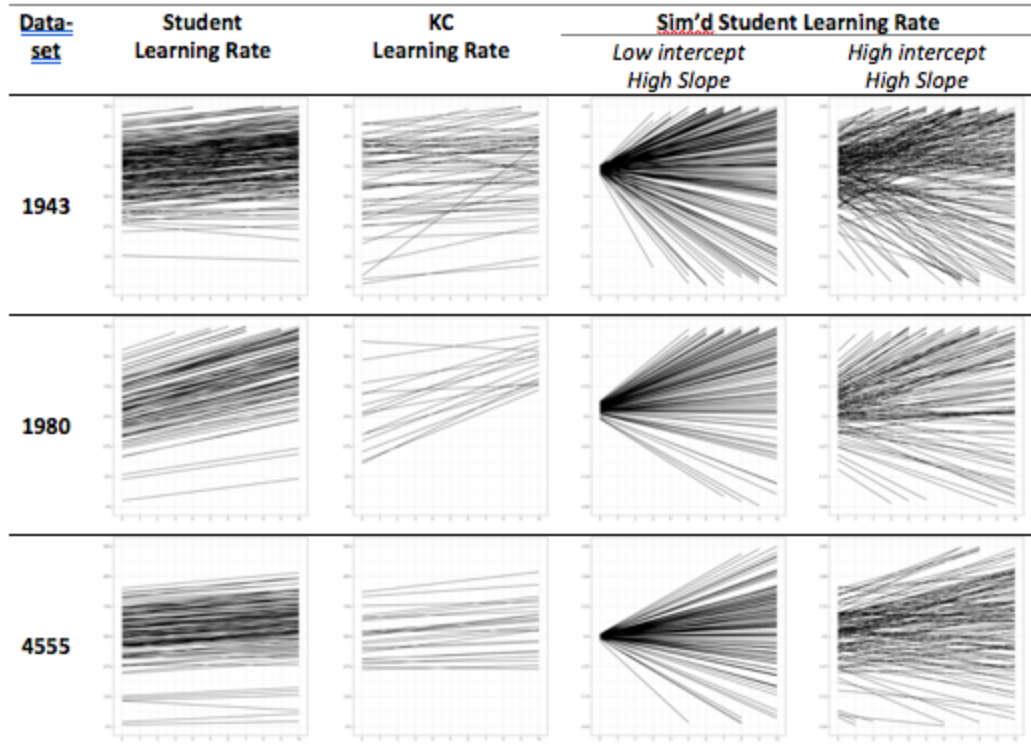


Fig. S10. Learning curves for 3 datasets with opportunities from 0-10 across the x-axis and percent correct on a log odds scale for the y-axis. Learning rate variation (slope) is relatively constant for students (near parallel lines in the first column) and higher for knowledge components (less parallel lines in the second column). The model can recover high student slope variation (non parallel lines) when present in simulated data whether the student intercept variation is low (column 3) or high (column 4).

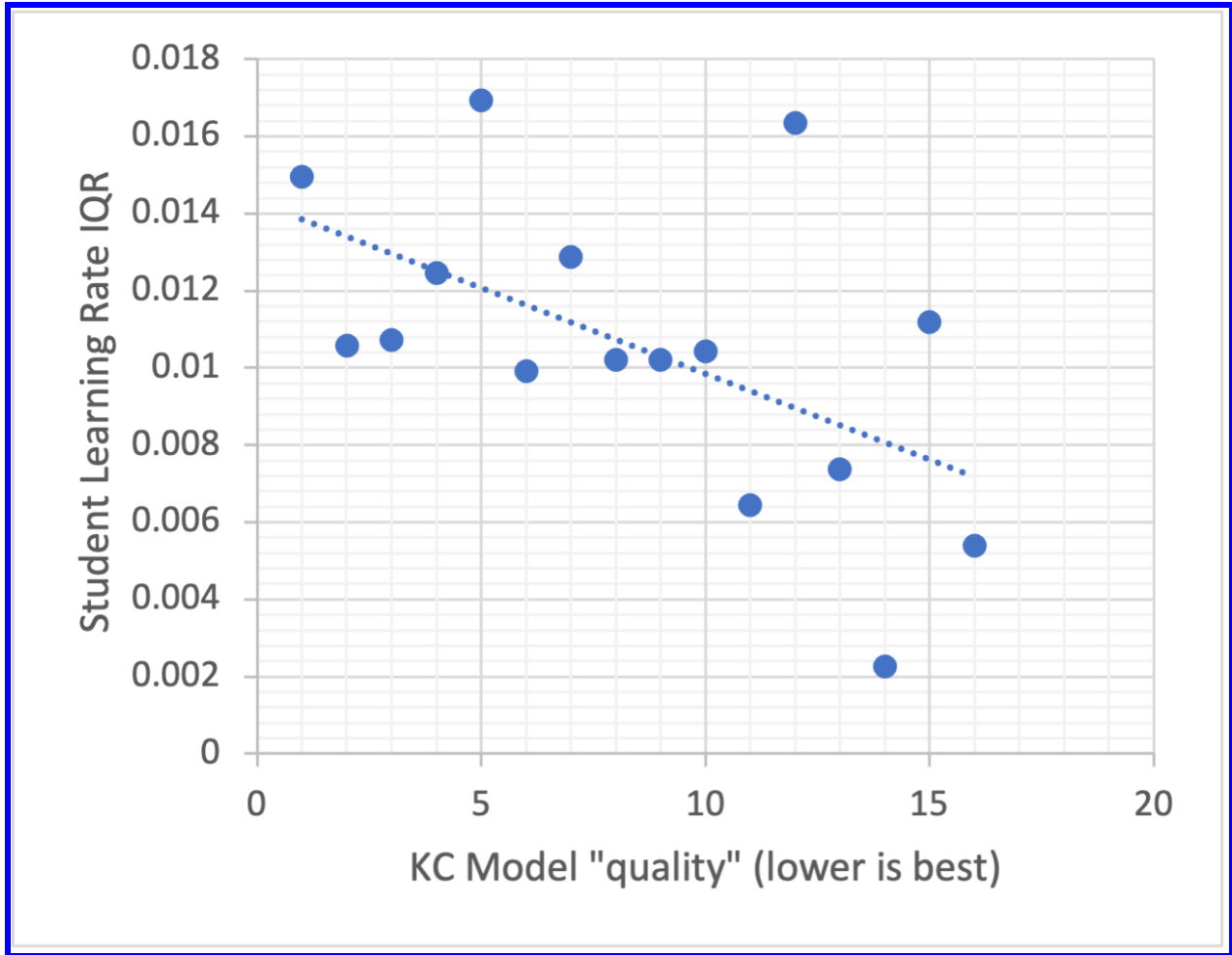


Fig. S11. Correlation between KC quality (0 is the best KC model, chosen for the analyses in the main text) and student learning rate variability (as measured by IQR).

Table S1. Dataset properties ordered by percent gain in learning rate for first opportunity.

Dataset	Domain	Approx Grade Level*	Ed tech	Stu	Obs	KC Models	KCs	KCs after exclusions	Obs/Stu-KC	Initial Knowledge (% accuracy)	Learning Rate (% gain for 1st opp)
ds115	Chinese	College	ITS	72	19,008	4	248	243	1.06	26.3%	19.9%
ds308	Statistics	College	Online	52	4,152	9	9	9	8.87	65.4%	9.5%
ds445	Fractions	Elem	Game	51	4,327	22	21	20	4.04	37.6%	7.1%
ds1387	Fractions	Elem	ITS	84	4,032	3	34	31	1.41	71.7%	5.8%
ds1980	English	College	Tutor	120	6,592	25	16	16	3.43	57.5%	3.6%
ds394	English	College	Tutor	97	5,773	12	13	13	4.58	68.6%	3.2%
ds406	CompSci	College	Online	765	441,643	5	74	69	7.80	65.4%	3.1%
ds1899	Algebra	Middle	Tutor	302	4,930	9	6	6	2.72	40.8%	3.0%
ds1935	STEM	College	Online	41	8,631	9	18	17	11.70	74.2%	3.0%
ds372	English	College	Tutor	99	7,128	16	19	19	3.79	73.3%	2.8%
ds567	Fractions	Elem	Tutor	59	48,501	21	236	194	3.48	68.5%	2.5%
ds99	Geometry	High	ITS	95	17,469	20	41	40	4.48	64.0%	2.5%
ds392	Geometry	Middle	ITS	123	41,756	8	38	38	8.93	78.5%	2.4%
ds565	Fractions	Elem	Tutor	61	57,948	7	78	78	12.18	62.9%	2.1%
ds447	Language	College	Tutor	161	92,067	4	46	46	12.43	65.7%	1.6%
ds1007	CompSci	College	ITS	49	5,063	9	4	4	25.83	55.6%	1.5%
ds566	Fractions	Elem	Tutor	58	64,025	9	60	58	18.40	71.5%	1.4%
ds562	Fractions	Elem	Tutor	63	48,739	21	102	98	7.58	61.5%	1.4%
ds563	Fractions	Elem	Tutor	64	55,407	17	54	54	16.03	66.3%	1.3%
ds104	Physics	College	ITS	104	6,069	6	13	12	4.49	76.6%	1.2%
ds271	Algebra	Middle	Tutor	69	1,103	5	6	6	2.66	33.8%	1.0%
ds1943	Geometry	Middle	ITS	218	136,354	4	64	63	9.77	71.1%	0.9%
ds253	Geometry	High	ITS	41	14,875	17	22	22	16.49	69.9%	0.8%
ds531	Language	College	Tutor	77	117,297	5	68	68	22.40	68.5%	0.8%
ds4555	Algebra	High	ITS	129	32,125	3	26	24	9.58	49.7%	0.8%
ds1330	Algebra	Middle	Tutor	3819	39,369	4	24	23	0.43	60.4%	0.7%
ds564	Fractions	Elem	Tutor	73	66,728	9	65	60	14.06	63.1%	0.3%
Mean				257	50,041	10	52	49	8.84	61.8%	3.1%
SD				725.7	84,662	6.78	60.62	55.29	6.69	13.3	3.94

Note: Approximate grade levels are Upper Elementary (4-6), Middle (6-8), High (9-12), College. Stu indicates the number of students in that dataset. Obs is the total number of unique opportunities in each dataset. KC Models is the number of KC models available in that dataset. Ed tech is the type of technology used in the dataset. KCs after exclusions indicates the number of KCs in the target KC Model after excluding KCs that do not satisfy the inclusion criteria (see text for details). Obs/Stu-KC is the number of steps per student-KC pair. Initial knowledge is the estimated baseline performance prior to practice in percent correct. Learning rate is the estimated increase in performance after the first opportunity to practice (converted from log odds to percent).

Table S2. Dataset descriptions and references to papers describing the datasets or, when references are followed by “related publication”, describing the educational technology used to generate the dataset. Blue highlights represent university courses or adult learners, red are high school courses, green are middle school courses, and purple are upper elementary classes (grades 4-6).

Dataset	Course	Description	Year	Reference
ds104	Physics	Intelligent tutoring system used in a college Intro Physics course at the US Naval Academy.	Spg 2006	1-3
ds1980 ds372 ds394	English as a Second Language	Tutoring system for students enrolled in a college English as Second Language course	Spg 2009 Fall 2009 Spg 2010	4
ds308	Statistics	Online interactive course (OLI) on Intro Statistics used in a college course	Spg 2009	5 (related publication)
ds1935	STEM Readiness	Online interactive course (OLI) for STEM Readiness used with college students	Spg 2013	5 (related publication)
ds406	Computing	Online interactive course (OLI) used in a computer literacy course for college Freshman	2010	5 (related publication)
ds1007	Spread-sheet	Tutoring system for adults writing spreadsheet formulas for problem scenarios	2002	6, 7
ds447 ds531	English as a Second Language	Intelligent tutoring system for English articles used by Chinese college students at a public university in Beijing	2010	8, 9
ds99	Geometry	Intelligent tutoring system used in a High School Geometry course	Winter 2007	10
ds253	Geometry	Intelligent tutoring system used in a High School Geometry course	2005-2006	11 (related publication)
ds605	Geometry	Intelligent tutoring system used in a High School Geometry course	Fall 2011	12
ds4555	Algebra	Intelligent tutoring system for writing equations from stories used in a High School course	2019	13
ds1899	Algebra	Online tutoring system for writing equations from stories used in an urban Middle School	2008-2009	14
ds271	Algebra	Online tutoring system for writing equations from stories used in an urban Middle School	Spg 2008	14 (related publication)
ds392	Geometry	Intelligent tutor for Geometry in Middle School	Spg 2010	15
ds1330	Math	Online math practice in 6th & 7th grade	2014-2015	16-17 (websites)
ds1943	Geometry	Intelligent tutor for Geometry used in a High School	2016-2017	18 (related publication)
ds1387	Fractions	Intelligent tutor system for fractions used in elementary schools	2013	19
ds445	Fractions	Educational game for fraction number line estimation used in elementary classes	2011	20
ds562-567	Fractions	Intelligent tutor system for fractions used in elementary schools	2011	21

Table S2. Chosen KC Model Name and link to data for each dataset used in the analyses.

Dataset	Chosen KC Model Name	Link to Data
ds99	LFA AIC LIB Model0	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=99
ds104	LFA Search	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=104
ds115	Response	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=115
ds253	LFASearchAICWholeModel1	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=253
ds271	Main-KC7-split	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=271
ds308	LFASearchModel1	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=308
ds372	Default corrected	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=372
ds392	Ktskills-Mcontext-single	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=392
ds394	LFASearchAICWholeModel1	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=394
ds406	LOv1.1	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=406
ds445	LFA AIC Model0 v2	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=445
ds447	Article Rule	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=447
ds531	Article Rule	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=531
ds562	new supportType topic	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=562
ds563	new topic	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=563
ds564	new topic	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=564
ds565	Default topic	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=565
ds566	new repr	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=566
ds567	Default reprType topic	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=567
ds1007	Skills4	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=1007
ds1330	CCSS standard	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=1330
ds1387	matched-Default	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=1387
ds1899	Main-KC7-split_renamed-PVfixed-models	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=1899
ds1935	LFASearchBICModel1-PVfixed-models uploaded	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=1935
ds1943	Default match	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=1943
ds1980	LFASearchModel0 Split	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=1980
ds4555	Refined both conditions	https://pslcdatashop.web.cmu.edu/LearningCurve?datasetId=4555

Table S4. Student learning rate variability (as measured by IQR) for the best KC model and a generally poor KC model (single KC).

Dataset	IQR Single KC	IQR Best KC Model
ds1007	0.008	0.047
ds104	0.007	0.012
ds115	0.002	0.102
ds1330	0.020	0.019
ds1387	0.003	0.048
ds1899	0.031	0.029
ds1935	0.001	0.001
ds1943	0.001	0.029
ds1980	0.010	0.024
ds253	0.001	0.015
ds271	0.011	0.064
ds308	0.003	0.007
ds372	0.006	0.074
ds392	0.002	0.013
ds394	0.009	0.054
ds406	0.000	0.008
ds445	0.010	0.018
ds447	0.001	0.036
ds4555	0.004	0.017
ds531	0.000	0.009
ds562	0.001	0.012
ds563	0.001	0.011
ds564	0.000	0.005
ds565	0.001	0.039
ds566	0.001	0.005
ds567	0.001	0.039
ds99	0.002	0.012

Table S5. Student learning rate variability (as measured by IQR) for High and Low ability students learning Easy and Hard KCs.

	Higher ability students	Lower ability students
Easy KCs	0.066 (M = 0.131, SD = 0.188)	0.034 (M = 0.081, SD = 0.143)
Hard KCs	0.040 (M = 0.058, SD = 0.081)	0.053 (M = 0.086, SD = 0.121)

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